Stephen Lumsden, essay for Units 6-9, Program C: The First Philosophers

(Amended Essay for Question 5. Analyse in detail any one of Zeno's paradoxes.)

Zeno and The Dichotomy Paradox

When I was a boy in school we were introduced to the idea of limits by an example of a frog jumping across a table. In its first jump it makes it half way across the table. It only has to get through the final half, but only gets halfway though that remaining distance. All its subsequent jumps will only get it through half the remaining distance. As a result it will never get to the other side of the table. It's limit will get quite near - so near in fact that the distance left will approach zero, but never get there. Years later I realise that this is just another development of Zeno's Dichotomy paradox, which we will examine in this essay. However, whereas the analogy of the frog jumping across the table may imply that it can never reach its goal, the Dichotomy Paradox implies it will never get started. How is this? We shall begin by summarising it, then highlight it's main points and then it's conclusions. We shall then review its main refutations and note some fallacies it may suffer from. We shall then finish, explaining its importance and conclude on its significance in philosophical debate today.

The Dichotomy paradox states that if you are going from point A to B, you first have to get to the half way point, say A1. Before that you have to get to the half way point to that, say A12. But before that you have to get half way to that point, and so on. There are so many points to get through it looks impossible to get to B, or any point along the route to it. You are not able to start traversing the length of A to B to any real extent. That also implies that movement is not possible. We could summarise the Dichotomy paradox as an argument as below:

> Premise 1: To reach his goal, a runner must touch infinitely many points ordered in the sequence 1/2, 1/4, 1/8, ...
> Premise 2: It is impossible to touch infinitely many points in a finite time.
> Conclusion: Therefore the runner cannot reach his goal.

However we know from real life this is untrue. One is tempted to dismiss this out of hand. There seems to be no real conclusive answers to this, but we are prompted to ask more questions. If we begin by looking at the above series and add them up:

1/2+1/4+1/8+1/16+1/32+...... = 1

We know this now through being able to do the reverse and divide a square up into a ½, the a ¼ and so on. What we are seeing in the above series in adding up all its elements is a convergent series, i.e. one which adds up, or converges, to a definite finite number. This stands in contrast to another series, namely the series of prime numbers, which diverges potentially to infinity (i.e. a divergent series).

Looking at the series above one is tempted to think that it is intuitive that the final steps will become negligibly small and become zero, but in fact we can never accept they will ever be zero. We may not touch infinitely many points as argued in premise 1 then and therefore cannot accept it.

These points which may approach zero, but will never be discrete points of zero which must be discretely traversed. Its is also, at this stage, useful to be familiar with the idea of denseness. This dictates that, between two points within of a certain distance A and B, there will always be space for another point. In this respect the space between A and B could be treated better as a continuous one, because if the infinite number of points between A and B did each have any actual size, this would add up to an infinite space which would be impossible to traverse, which we know it is not. These examples relate to what Aristotle defined as potential vs actual cases of infinity. In the case of a potential infinity Aristotle allowed we could traverse the potentially infinite points between A and B, but never if there were an actual infinity of them. So the second premise of the argument could also be refuted.

We do not even have to look so deeply into the main points of the argument to refute it though. Even from a logical standpoint it fails as its falls prone to several different fallacies and contradictions. From another paradox, namely Achilles and the Tortoise, we can see Zeno contradicts himself, because he sets up a condition where Achilles can never catch up with the tortoise in a race, despite the fact that Zeno is trying to prove movement is impossible in the Dichotomy paradox! Secondly the fallacy of false cause implies that Zeno gives us no reason from being only able to get half way from A to B, but merely makes this assertion and concludes it we cannot get to the destination, movement is not possible. Additionally the fallacy of non-sequitar implies Zeno's positive premises are getting negative results, which does not stack up.

At this stage we may be tempted to ask why be interested in a philosophical argument which is so easy to refute. Surely if rational argument can break such paradoxes down, what is their use? Could we look at apparently similar series (to 1/2+1/4+1/8+1/16+...) and make similar conclusions about them. For instance if we looked at another series of rational numbers

1/2 + 1/3 + 1/4 + 1/5 + 1/6 + 1/7 + …...

could we assume this will be convergent also? This may have been a logical proposition at one stage,but we know from since the seventeenth century that the above series is divergent and will go on forever and there will be no limit to its partial sums . Therein lies one significant aspect of Zeno's Dichotomy paradox. It make us look deeper, not trust one's senses fully, but try to use reason and build on the one's rationale. It may be in raising such paradoxes and their unlikely outcomes, that this method of reductio absurdum (using an argument to arrive at an absurd conclusion) helps us to distinguish what is correct. This may be countered by the view that Zeno's arguments are merely used as a defence of Parmenides' arguments against plurality and motion, where perhaps such defences are too contrived to be practical. One could be tempted to ask whether Zeno interpreting Parmenides idea of the 'One' too strictly? Such scenarios are useful though, as the author believes there may be an element of showmanship on Zeno's part or intended provocation, similar to Heraclitus' arguments, in coercing the reader to use their own reason. In the absence of what we know now due to the mathematics of Newton, Cantor and Leibniz, such basic speculation had to be a necessary forerunner to real scientific inquiry.

We have examined the Dichotomy paradox as a straight syllogism while concentrating on its premises. It clearly fails, but the defence of Parmenides' theories is not its main purpose. The method of arguing reductio absurdum is a strong one and will prompt further enquiry in even the most indifferent minds, once the conclusion cannot be accepted. It prompts us to question the nature and qualities of time, space and identity, right from the philosopher down to the school child being introduced to the basic mathematical concepts we take for granted today.

References

The Presocratic Philosophers (Cambridge University Press), by G.S. Kirk, J.E. Raven and M. Schofield, 1983

The Greek Philosophers: From Thales to Aristotle (University Paperbacks), by W.K.C Guthrie, 1968

Early Greek Philosophy (Penguin Classics), by Jonathan Barnes, 2001

Space And Time (Routledge, 2 edition) ,by Barry Dainton, 2010

Ask A Philosopher Q and A:

https://askaphilosopher.org/2015/05/19/parmenides-and-zeno-on-the-impossibility-of-change-and-motion/

https://askaphilosopher.org/2017/11/28/is-achilles-still-chasing-that-tortoise/

Zenos's Paradoxes: Achille and The Toroise and the Dichotomy, George Walendowski

https://philosophypathways.com/essays/walendowski2.html